Unfolding Neural Networks for Compressive Multichannel Blind Deconvolution

Bahareh Tolooshams[*1], Satish Mulleti[*2], Demba Ba[1], and Yonina C. Eldar[2]

[1]Harvard University [2]Weizmann Institute of Science [*]Equal contributions

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CRISP

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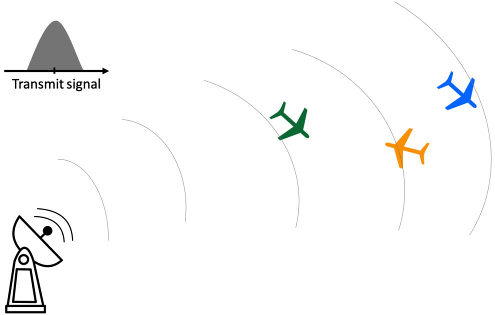
SAMPLLAB
Signal Acquisition Modeling Processing and Learning

Motivation

A transmit source signal is reflected from *sparsely* located targets and measured at the receiver.



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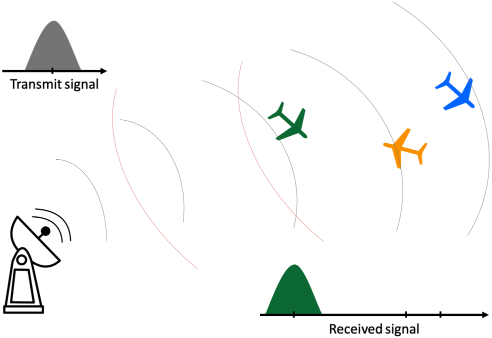

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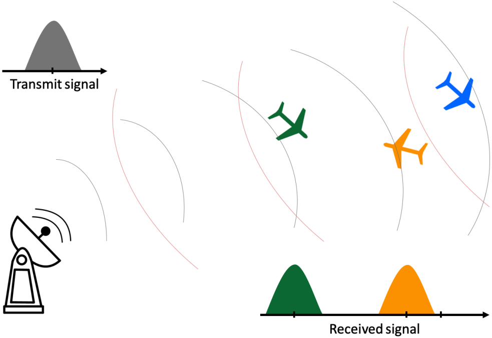

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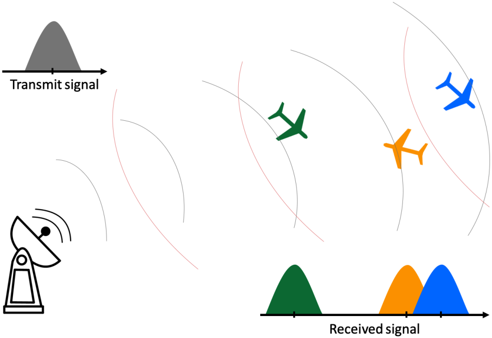

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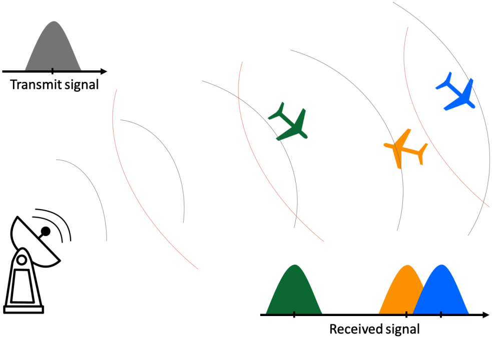

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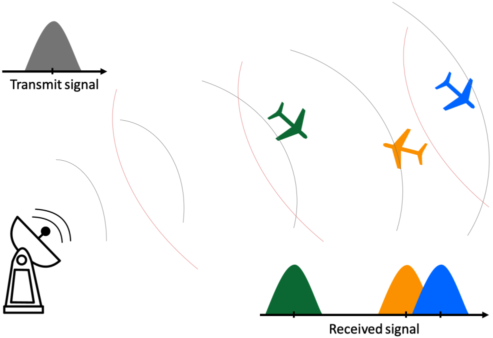

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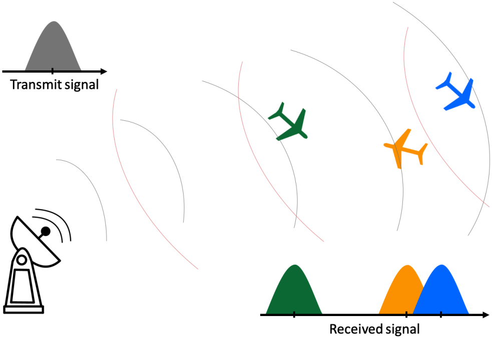

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Challenges: Receivers' complexity increases with number of measurements.

Goal: Design a *hardware-efficient* and *data-driven* compression to enable recovery from compressed measurements.

Given $n = 1, \ldots, N$ receiver channels,

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One way to solve:

$$\min_{\mathbf{s}, \{\mathbf{x}^n\}_{n=1}^N} \sum_{n=1}^{N} \frac{1}{2} \|\mathbf{y}^n - \mathbf{C}_s \mathbf{x}^n\|_2^2 + \lambda \|\mathbf{x}^n\|_1 \quad \text{s.t. } \|\mathbf{s}\|_2 = 1$$

Multichannel Blind Deconvolution (MBD)

Sparse-MBD

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Challenges:

- Requires access to full measurements $\mathbf{y}^n$.
- Computationally demanding.

Recover $\mathbf{s}$ and $\mathbf{x}^n$ from *compressive* measurements $\mathbf{z}^n = \mathbf{\Phi}\mathbf{y}^n$.

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Prior works:

Compressive sparse-MBD

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- Pick $\mathbf{\Phi}$ as a random matrix [1]:
  - fast ✓, not hardware-efficient ✗

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Prior works:

- Pick $\mathbf{\Phi}$ as a random matrix [1]:
  - fast ✓, not hardware-efficient ✗
- Design a *structured* $\mathbf{\Phi}$ [2]:
  - slow ✗, hardware-efficient ✓

Our Contributions

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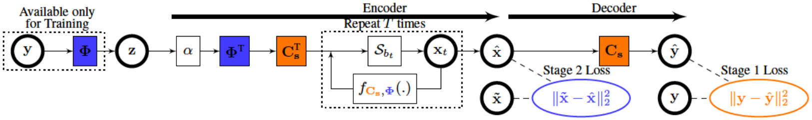
Learned **S**tructured compressive **M**ultichannel **B**lind **D**econvolution



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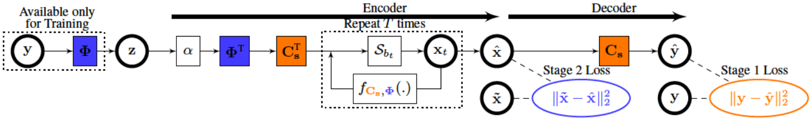

- An unfolding neural network

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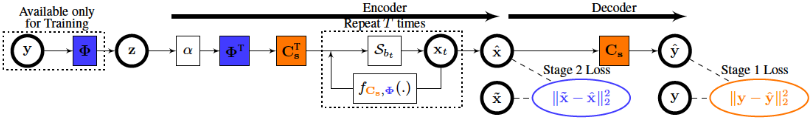

- An unfolding neural network
  - Fast and computational efficient ✓

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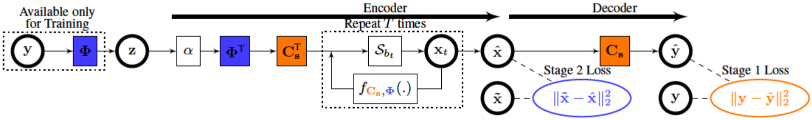

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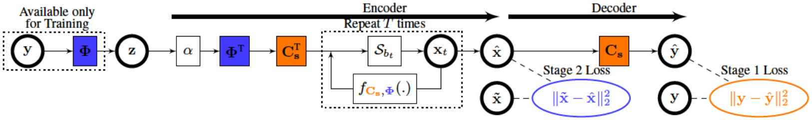

- An unfolding neural network
  - Fast and computational efficient ✓
- Learned structured compression
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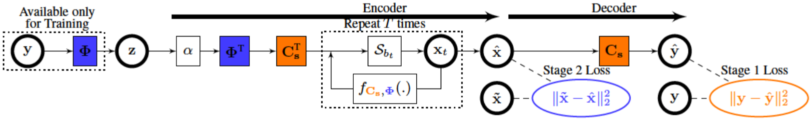

- An unfolding neural network
  - Fast and computational efficient ✓
- Learned structured compression
  - Hardware-efficient ✓
  - Superior performance against prior works ✓

LS-MBD

Compression operator

Compress through a *convolution* followed by a *truncation*.

$$\mathbf{z} = \lfloor \mathbf{h} * \mathbf{y} \rfloor_{\mathsf{trunc}} = \mathbf{\Phi y}$$

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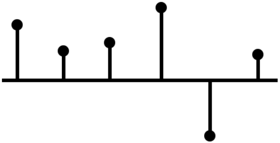
Full Measurements



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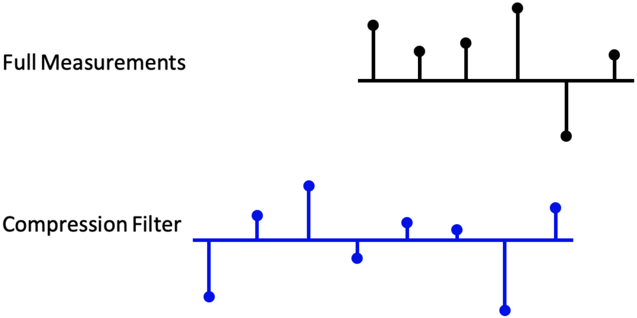

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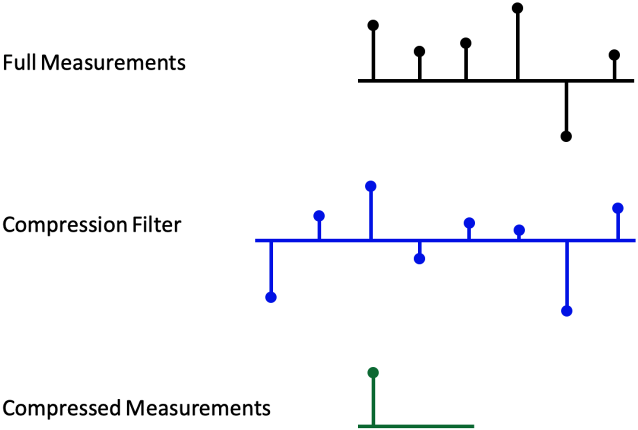

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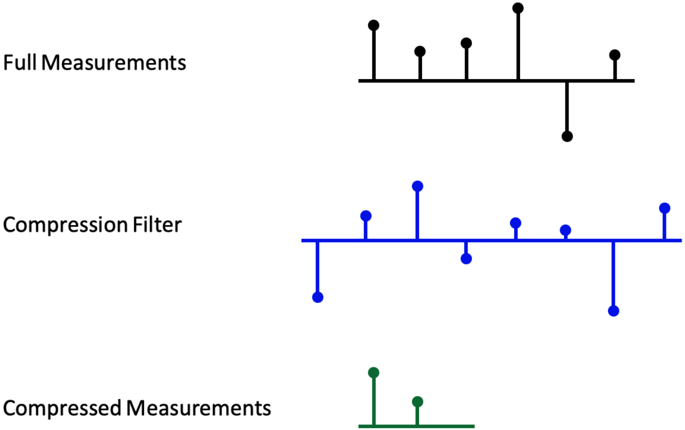

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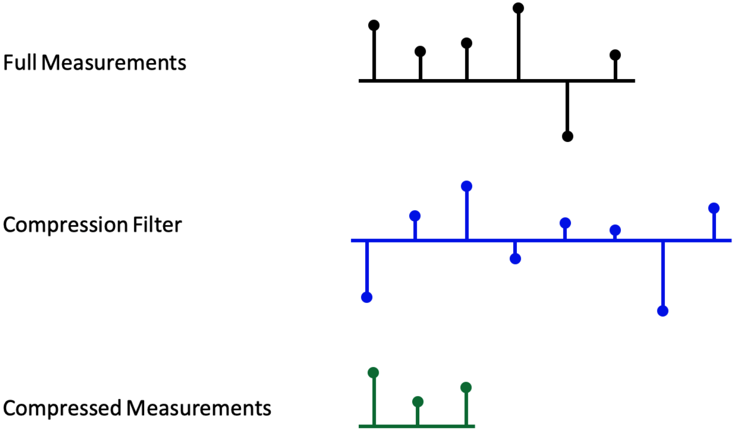

LS-MBD

Network architecture

$$\min_{\mathbf{\Phi},\mathbf{s},\{\mathbf{x}^n\}_{n=1}^N} \sum_{n=1}^N \frac{1}{2}\|\mathbf{z}^n - \mathbf{\Phi}\mathbf{C}_s\mathbf{x}^n\|_2^2 + \lambda\|\mathbf{x}^n\|_1 \quad \text{s.t. } \|\mathbf{s}\|_2 = \|\mathbf{h}\|_2 = 1$$

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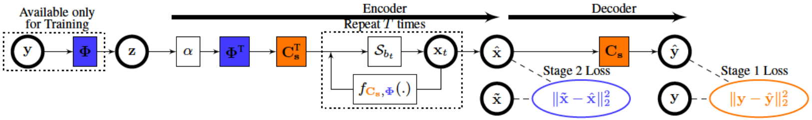

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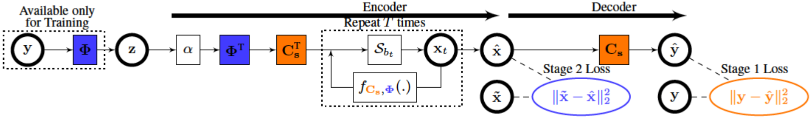

Unfolding neural network:

LS-MBD

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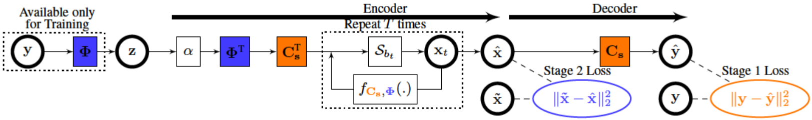

Unfolding neural network:

- Encoder: proximal gradient descent to map compressed measurements $\mathbf{z}^n$ to target locations $\mathbf{x}^n$.

LS-MBD

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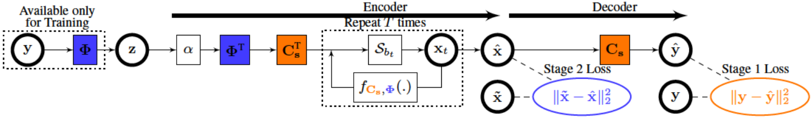
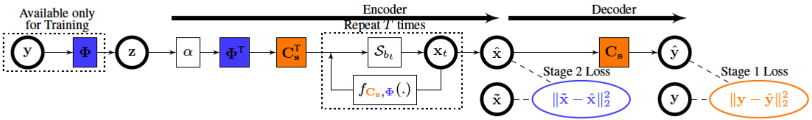


Unfolding neural network:

- Encoder: proximal gradient descent to map compressed measurements $\mathbf{z}^n$ to target locations $\mathbf{x}^n$.
- Decoder: use the source $\mathbf{C}_s$ to reconstruct full measurements $\mathbf{y}^n$.

LS-MBD

Training



LS-MBD

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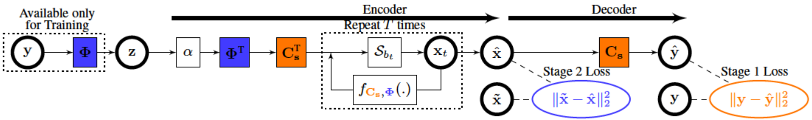

Stage 1:

LS-MBD

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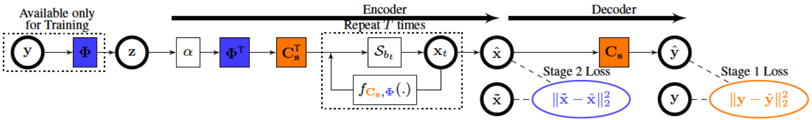

Stage 1:

- Train with full measurements to recover source $\mathbf{C}_s$ (i.e., set $\mathbf{\Phi} = \mathbf{I}$).

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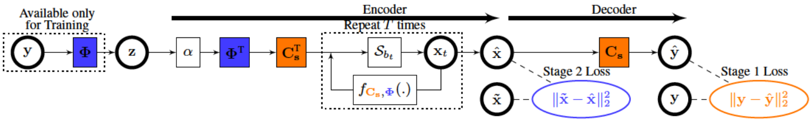

Stage 1:

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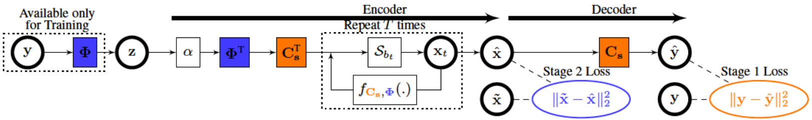

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LS-MBD

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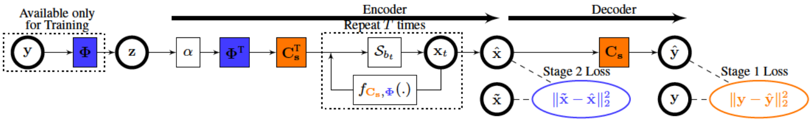

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Stage 2:

LS-MBD

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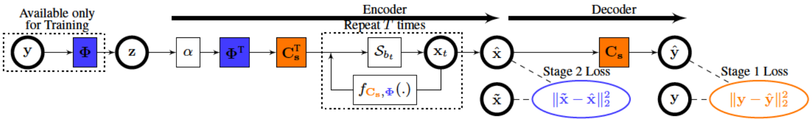

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Stage 2:

- Take estimated codes $\tilde{\mathbf{x}}$ and source $\mathbf{C}_s$ from stage 1.

LS-MBD

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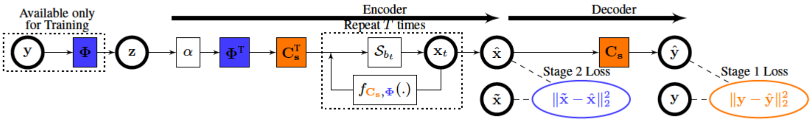

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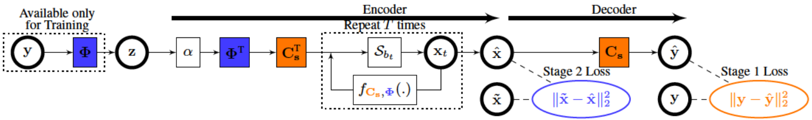

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Stage 2:

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- Forward pass: Estimate code & compute loss function $\|\tilde{\mathbf{x}} - \hat{\mathbf{x}}\|_2^2$.
- Backward pass: Learn compression $\mathbf{\Phi}$.

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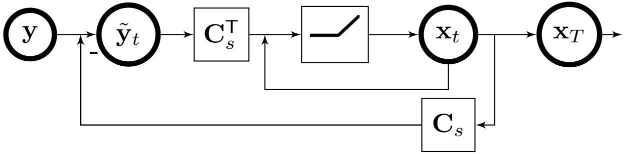
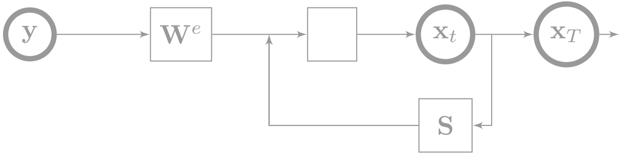
Solve sparse coding by iterative proximal gradient algorithm.

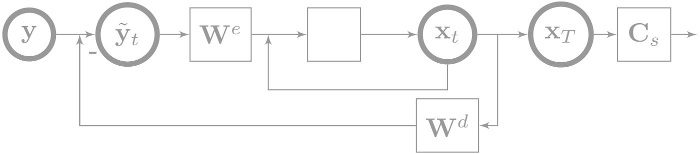
ISTA [3]:



LISTA [4]:



CRsAE [5]:



Prior Works on Unfolding Networks

Solve sparse coding by iterative proximal gradient algorithm.

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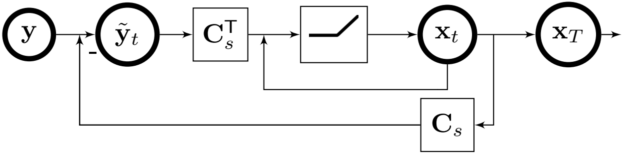
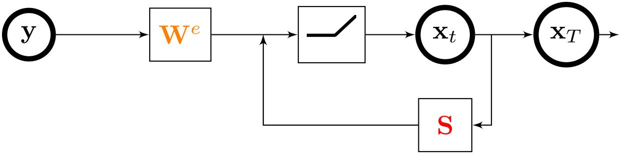
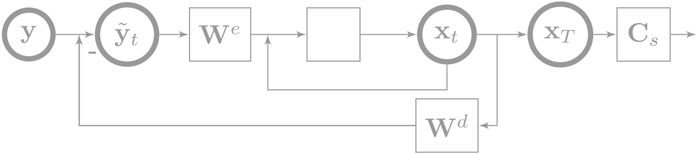


LISTA [4]:



CRsAE [5]:



Prior Works on Unfolding Networks

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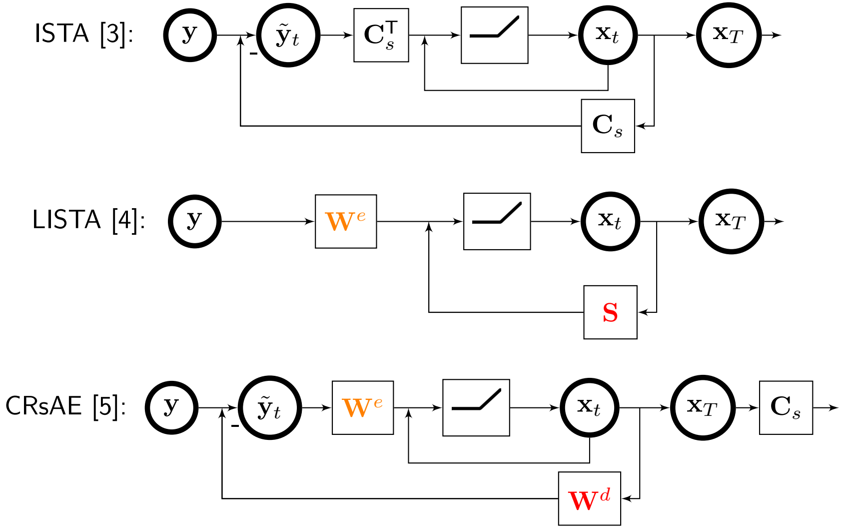
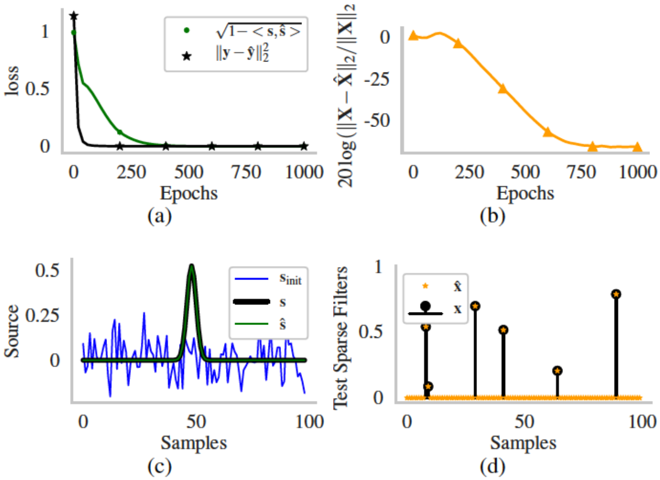


Results

Recovery performance (I)



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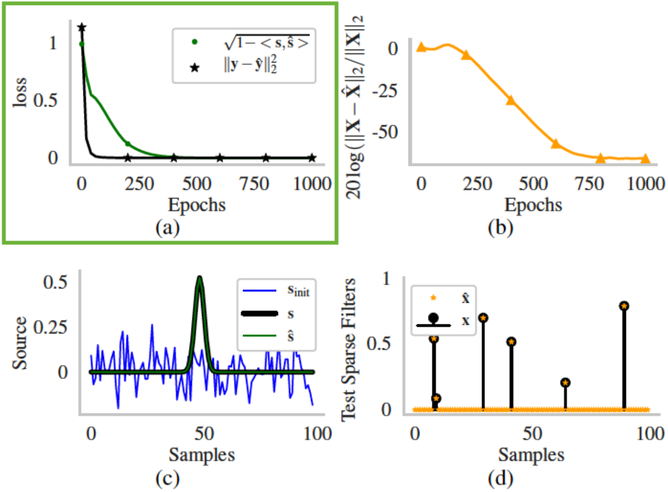

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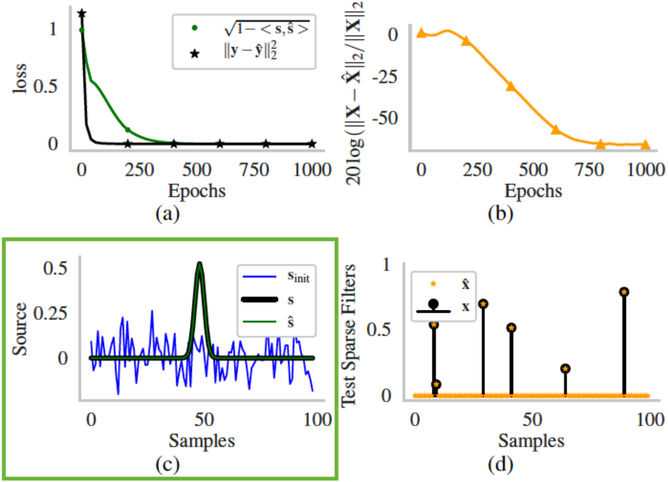

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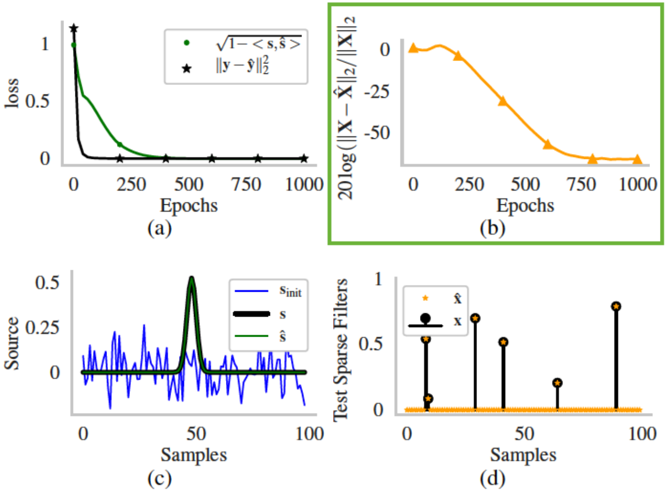

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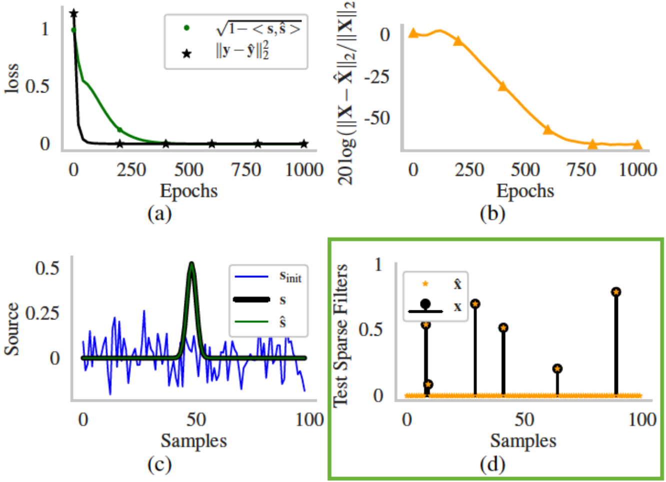

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- **LS-MBD**: $\mathbf{\Phi}$ is learned and structured.
- **LS-MBD-L**: $\mathbf{\Phi}$ is learned and structured (relaxed network as in LISTA).
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Sparse code recovery error

CR [%]	M_z	G-MBD	GS-MBD	FS-MBD	LS-MBD	LS-MBD-L
50	99	**-54.05**	-44.93	-43.96	**-53.27**	-26.54
40.4	80	**-55.07**	-40.55	-26.52	**-52.80**	-
35.35	70	**-52.43**	-40.00	-22.76	**-51.50**	-
31.31	62	**-53.63**	-37.13	-21.86	**-54.71**	-
25.25	50	**-53.36**	-28.57	-8.40	**-51.41**	-
23.74	47	**-50.60**	-26.11	-6.84	**-50.35**	-
22.72	45	**-52.98**	-23.17	-6.14	-43.61	-
20.20	40	-47.39	-14.75	-5.13	-17.07	-

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Method \ Cost	Memory Storage	Complexity
Structured	$O(M_h)$	$O(M_h \log M_h)$
Unstructured	$O(M_y M_z)$	$O(M_y M_z)$

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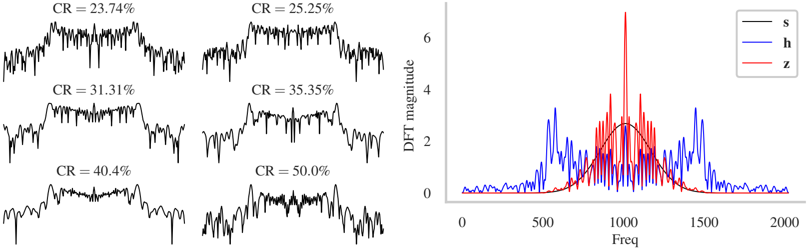
Unstructured	$O(M_y M_z)$	$O(M_y M_z)$

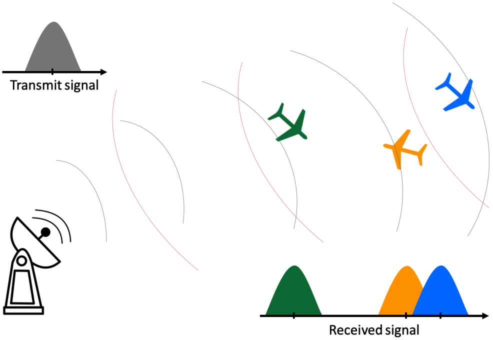
Speed \ Method	G-MBD	FS-MBD	LS-MBD	LS-MBD-L
runtime [s]	4.9087	164	5.4204	**0.0028**

Results

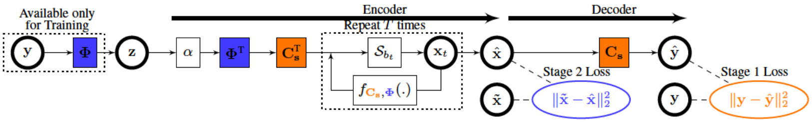
Compression visualizations



Conclusion



Transmit signal
Received signal



Available only for Training
Encoder
Decoder
Repeat T times
Stage 2 Loss
Stage 1 Loss

References

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B. Tolooshams, S. Dey, and D. Ba, "Deep residual autoencoders for expectation maximization-inspired dictionary learning," *IEEE Trans. Neural Netw. Learn. Syst.*, pp. 1–15, 2020.