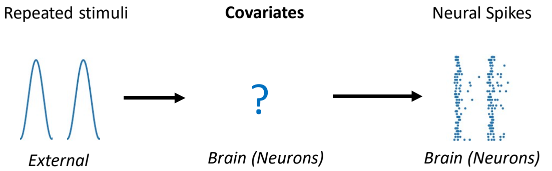
# Convolutional Dictionary Learning of Stimulus from Spiking Data

Andrew Song[1*], Bahareh Tolooshams[2*], Simona Temereanca[3], Demba Ba[2]
(1): Dept. of Electrical Engin. & Computer Sci., MIT, Cambridge, MA; (2): School of Engineering and Applied Sciences, Harvard University, Cambridge, MA;
(3): Brown University, Providence, RI; (*): Equal Contributions

## Problem Statement

The recording of neural activity in response to repeated presentations of an external stimulus is an established experimental paradigm in neuroscience. GLMs are commonly used to describe how neurons encode external stimuli, by statistically characterizing the relationship between the covariates and neural activity. The question becomes: *How do we choose appropriate covariates*?




We cast the problem of *learning the covariates* from the data as a **Convolutional Dictionary Learning** (CDL) problem, where the goal is to learn 1) shift-invariant templates from multiple sources and 2) the times when they occur. Our approach is ***data-driven*** and thus ***unsupervised***.

Our contributions are
- Formulate optimization objective with ***sparsity constraints***, also accounting for binary nature of spikes
- Propose iterative algorithms to solve the objective, with the key insight that observations need to be iteratively modified

## Generative Model

### Notations

$y^{j,m} \in \{0,1\}^N$: Spike time-series for Neuron $j$ and trial $m$
$h_c \in \mathbb{R}^K$: Template from source $c$
$X_c^j \in \mathbb{R}^{N-K+1}$ : Code vector for neuron $j$ and source $c$
$a_j$: Baseline firing rate for neuron $j$

### Generative Model

$y^j \sim Binomial(M_j, \mu_j)$, where $\mu_j = a_j + \sum_c^C h_c * X_c^j$

### Optimization objective

$$\min_{h_c, X_c^j} \sum_j^J \sum_m^{M_j} -\left(a_j + \sum_c^C h_c * X_c^j\right)^T y^{j,m} + 1_N^T \log\left(1 + \exp\left(a_j + \sum_c^C h_c * X_c^j\right)\right)$$

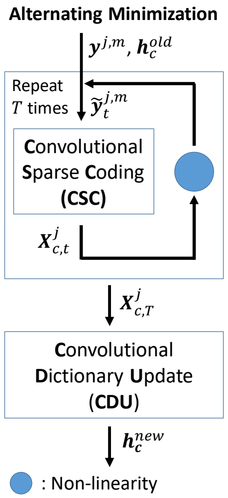
Such that $X_c^j \geq 0$ and $X_c^j$ is sparse

### Approach

- We use alternating minimization (or block coordinate descent) to solve for $\{h_c\}_c$ and $\{X_c^j\}_{c,j}$, in an alternative manner.
- To account for **binary-nature** of the data, we *iteratively modify* $y^{j,m}$

## Algorithms

Different sparsity constraints → Different CSC algorithms




### Modified observations

$$\tilde{y}_t^{j,m} = y^{j,m} - \left(1 + \exp(-a_j - \sum_c^C h_c * X_{c,t}^j)\right)^{-1}$$

- The modification is inspired by the *Iterative Reweighted Least Squares* algorithm for GLM

### Two algorithms

**1. $l_0$ pseudo-norm: $\|X_c\|_0$ (BCOMP)**

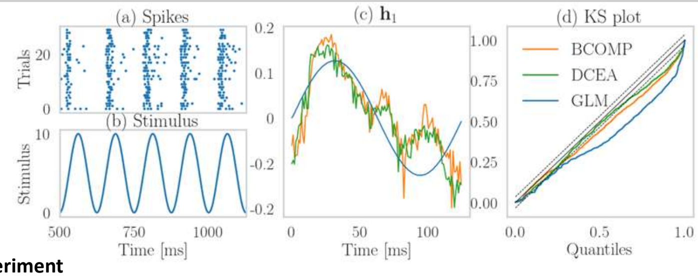
**CSC:** Extension of **Kernel greedy pursuit** to convolutional setting
**CDU**: Convolutional K-SVD

**2. $l_1$ norm: $\|X_c\|_1$ (DCEA)**

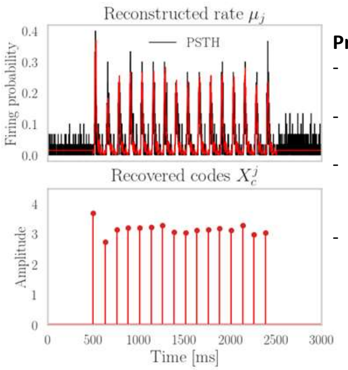
**CSC**
- Proximal gradient update (Guaranteed convergence)
- Map $T$ iterations of proximal update steps to $T$ layer recurrent neural network (efficiency + scalability)

**CDU**
- Backpropagation through the entire network

## Real Data – Barrel Cortex

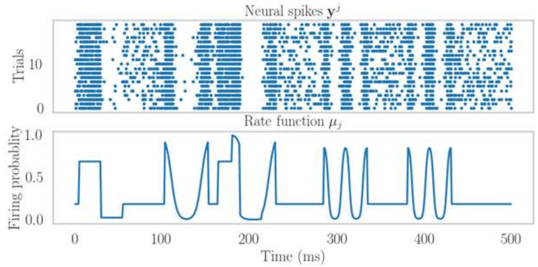



**Experiment**
- A whisker is repeatedly (16 times) stimulated with the same stimulus by piezo-electrode, inducing responses in Barrel cortex neurons.

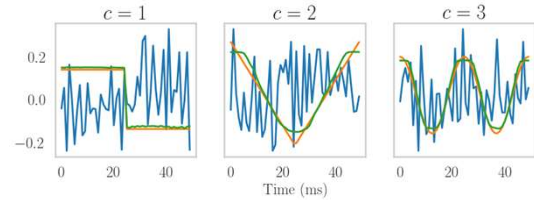



**Preliminary results**
- Previous GLM analysis requires ***hand-crafted*** or ***parametric function*** derived from the stimulus
- The ***learned templates*** (orange, green) have multiple peaks and different from the ***ideal whisker velocity*** (blue)
- The goodness-of-fit via KS plot shows that the learned templates explain the data better (closer to 45-degree line)
- For the test data, our framework *accurately identifies the onset times* (red) of 16 stimuli.

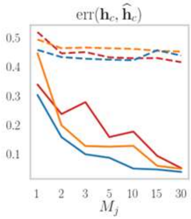
## Simulation Study




An example of neural spikes and the rate (truth) with $a_j = -1.5$




Random initialization for 3 templates (Blue), true templates (Orange), and learned templates after training (Green) → **CDL success**




Iteratively modifying observations is important
- **Solid** *with* modification
- **Dashed** *without* modification (Gaussian)

→ Accounting for binary nature of the data is ***crucial*** for learning the true templates

## Discussion & Future Work

**Benefits of our approach:**
- Unsupervised approach for discovering optimal covariates for explaining the data
- Accurate identification of the time occurrence

**Future work:**
- Validation with more behavioral datasets
- Add additional relaxations (onset delays across neurons) and constraints (learned covariates are smooth)

**References and Related Work:**
[1] Garcia-Cardona C., *IEEE TCI*, 2018
[2] Mackevicius E.L et. al., *eLIFE*, 2019
[3] Temereanca S., *J. Neuroscience*, 2008

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